# The Properties of Fuzzy Green Relations on Bilinear Form Semigroups

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*Abstract*—**The Green relations on semigroups have been introduced by Howie [3]. They are right Green relation $\mathcal{R}$, left Green relation $\mathcal{L}$ and (two sided) Green relation $I$. The right Green relation $\mathcal{R}$ is defined as $\{(x,y)\in S\times S|\langle y\rangle_R\}$, with $\langle x\rangle_R$ denotes the right ideal generated by an element $x$ (or called the principle right ideal generated by $x$. The definition of the left Green relation L and the Green relation $\mathcal{J}$ are similar to the definition of the right Green relation. In this paper we will construct the definition of the fuzzy right Green relation (denoted by $\mathcal{R}^F$), the fuzzy left Green relation (denoted by $\mathcal{L}^F$) and the fuzzy Green relation (denoted by $\mathcal{J}^F$) on a semigroup. First we define a fuzzy ideal (right/left) generated by a fuzzy subset (a fuzzy principle ideal) on a semigroup and their examples. Based on the fuzzy principle ideal definition, we define a fuzzy (right/left) Green relation on a semigroup. The fuzzy subset $\mu$ and $\rho$ are fuzzy (right/left) Green related if and only if the fuzzy (right/left) ideal generated by $\mu$ is equal to the fuzzy (right/left) ideal generated by $\rho$.**

***Keywords—Green relation, fuzzy ideal, fuzzy pricipal ideal, fuzzy Green relation***

## I. Introduction

A non empty subset $I$ of a semigroup $S$ is called a right (left) ideal if $IS\subseteq I$ $(SI\subseteq I)$ and an ideal (two sided) if $I$ is both a right ideal and a left ideal. The right (left) generated by $x\in S$ is denoted by $\langle x\rangle_R$ $(\langle x\rangle_L)$ and an ideal generated by $x\in S$ is denoted by $\langle x\rangle$. The Green relation on a semigroup has been introduced by Howie [3]. They are right Green relation (R), the left Green relation (L ) and the Green relation (I) . The green relation $\mathcal{R},\mathcal{L},\mathcal{J}$ are equivalence relations, defined as follow:

$$\mathcal{R}=\{(x,y)\in S\times S\ |\langle x\rangle_R=\langle y\rangle_R\}$$

$$\mathcal{L}=\{(x,y)\in S\times S\ |\langle x\rangle_L=\langle y\rangle_L\}$$

$$\mathcal{J}=\{(x,y)\in S\times S\ |\langle x\rangle=\langle y\rangle\}$$

Some papers related to the fuzzy ideal of semigroups, the fuzzy ideal of semigroups generated by a fuzzy singleton and their properties have been introduced by Karyati [5]. In this paper we will discuss how to define the fuzzy Green relations on a semigroup based on the fuzzy (right/left) ideal generated by a fuzzy subset of this semigroup.

## II. Fuzzy Green Relations on semigroup

Refer to Asaad [2], Kandasamy [4], Mordeson and Malik [7], a fuzzy subsemigroup $\mu$ of a semigroup S is defined as a mapping from $S$ into the interval [0,1], i.e. $\mu:S\to[0,1]$ which fulfils the condition $\mu(xy)\geq\min\{\mu(x),\mu(y)\}$ for all $x,y\in S$. A fuzzy subset $\mu$ is called a fuzzy right ( fuzzy left ) ideal of $S$, if for every $x,y\in S$ then $\mu(xy)\geq\mu(x)($ $\mu(xy)\geq$ $\mu(x)$ $)$ and $\mu$ is called fuzzy ideal of $S$ if $\mu$ is both a fuzzy right ideal and a fuzzy left ideal, i.e. $\mu(xy)\geq\max\{\mu(x),\mu(y)\}$ for all $x,y\in S$. Fuzzy subsets $\curlywedge$ and $\mu$ are called $\curlywedge\subseteq\mu$ if and only if $\curlywedge(x)\leq\mu(x)$ for every $x,y\in S$. A fuzzy relation $\theta$ of $S$ is defined as a mapping from $S\times S$ into the closed interval [0,1].

**Definition 2.1.** ([1], [6], [9]) *Let S be a semigroup and $\mu$ be a fuzzy relation on S. Then*

*(1) A fuzzy relation $\mu$ on $S$ is said to be reflexive if $\mu(x,x)=1$ for all $x\in S$*
*(2) A fuzzy relation $\mu$ on $S$ is said to be symmetric if $\mu(x,y)=\mu(x,y)$ for all $xx,y\in S$*
*(3) If $\mu_1=\mu_2$ are two relations on S, then their max-product composition denoted by $\mu_1\circ\mu_2$ is defined as*

$$\mu_1\circ\mu_2(x,y)=\max_{z\in S}\{\mu_1(x,z),\mu_2(z,y)\}$$

*(4) If $\mu_1=\mu_2=\mu$ and $\mu\circ\mu\leq\mu$, then the fuzzy relation $\mu$ is called transitive.*

Refer to Aktas [1], Kuroki [6], and Murali [9], we give some kinds of relations defined as follow :